The 2D 'Shock-Jet' Problem

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Introduction

Numerical simulation of Richtmyer-Meshkov instability (RMI) is conducted on a cylindrical interface between air and SF_6 accelerated by a Mach 1.2 shock initially in air. Computational setup is intended to match conditions published by Shankar, Kawai, & Lele [1] in their simulation of experimental work by Tomkins *et al* [2].

Computational Domain

The mesh is Cartesian and planar, parameterized by the diameter of the SF_6 cylinder (D=0.6cm). The center of the cylinder is located at the origin, and the mesh extends from -17D to 38D in the streamwise (x) dimension and from -12.5D to 12.5D in the stream-normal (y) dimension, as illustrated in figure 1. **Note: If non-reflecting boundary conditions are not available, these dimensions will need to be increased to prevent reflected waves from affecting the cylinder.** Grid spacing should be defined by the number of zones across one cylinder diameter. Suggested values: 82, 245, and 736 zones/cylinder. Use of adaptive mesh refinement (AMR) is recommended to localize zones around the cylinder and shock interface.
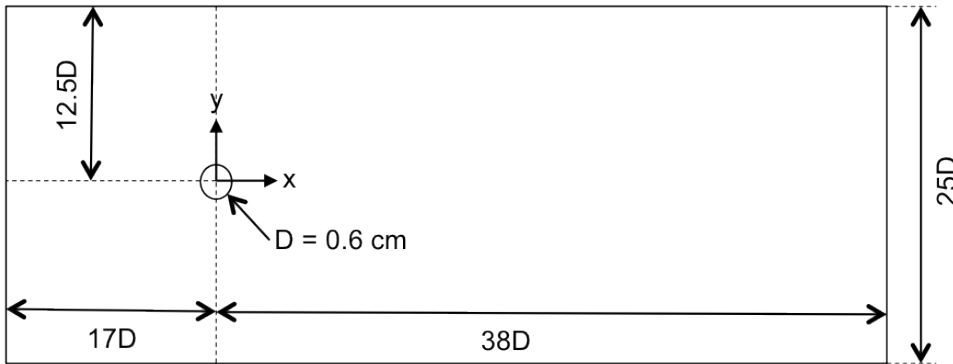

Figure 1. Computational domain (not to scale).

Initial Conditions

The problem is solved in a laboratory fixed coordinate frame such that the shock, initially located at x=-1.5D, is observed to propagate from left to right at a speed of Mach 1.21. Initial pressure and temperature in the stationary unshocked gas (right of the shock wave) are 0.8atm and 298K, respectively. Initial velocity and thermodynamic conditions in the shocked gas (left of the shock wave) are given by the Rankine-Hugoniot jump conditions. Air is assumed to be a mixture of 79% N_2 and 21% O_2 (by molar ratio). Initial mass fraction of SF_6 within the cylinder is given as a function of radial distance, r, from the origin where $Y_{max}^0 = 0.83$.

$$Y_{SF_6}^0(r) = \begin{cases} Y_{max}^0 \left(1.0 - \exp\left(\frac{-\left\|\left[\left(\frac{|r|}{D/2} - 1\right)\pi\right]\right\|^{1.54}}{1.0082} \right) \right) & |r| \leq \frac{D}{2} \\ \\ 0.0 & |r| > \frac{D}{2} \end{cases}$$

Equations of State

Thermodynamic variables in both air and SF$_6$ are assumed to be governed by the ideal gas equation of state, with the ratio of specific heats (γ) equal to 1.40 for air and 1.09 for SF$_6$.

Boundary Conditions

Inflow conditions should fix velocity and thermodynamic conditions in the shocked gas. Outflow conditions should fix the gas velocity, with special care taken not to reflect the principal shock wave. North and south boundaries should be non-reflecting "sponge" boundaries to allow perturbations in velocity and thermodynamic variables to go smoothly to zero. **If non-reflecting boundary conditions are not available, the computational mesh will need to be extended in both the streamwise and stream-normal dimensions to prevent reflection of the primary shock and secondary acoustics from interacting with the cylinder.**

Temporal Evolution

The problem is initialized at time $t = 0$ and run out to $t = 1000\ \mu s$.

Diagnostics

The following data are suggested for comparison:
1. 2D fields of SF$_6$ concentration (mass fraction) in pseudo-color at times $t = 130, 220, 310, 400, 490, 560, 650, 760, 880,$ and $1000\ \mu s$.
2. 1D plots of total mixing rate (TMR) versus time.
3. 1D plots of enstrophy (Ω) versus time.

TMR is an integral measure of the instantaneous scalar dissipation rate (or mixing rate), defined below, where $D_{SF_6}^{mol}$ and $D_{SF_6}^{turb}$ represent, respectively, the molecular and turbulent coefficients of diffusivity for SF$_6$ into air.

$$TMR = \iint \left(D_{SF_6}^{mol} + D_{SF_6}^{turb} \right) \nabla Y_{SF_6} \cdot \nabla Y_{SF_6}\, dxdy$$

For the Tipton K-L model, $D_{SF_6}^{turb} = \mu_t / (\rho Sc)$. If molecular viscosity and diffusion are computed, $D_{SF_6}^{mol}$ is given by the Chapman-Enskog method [1]. Otherwise, $D_{SF_6}^{mol} = 0$.

Enstrophy is an integral measure of total vorticity, defined below as the integral of the square of vorticity.

$$\Omega = \iint \omega_z^2\, dxdy$$

References

[1] Shankar, S.K., Kawai, S., and Lele, S.K. "Two-dimensional viscous flow simulation of a shock accelerated heavy gas cylinder," *Physics of Fluids*, 23(2), 2011.
[2] Tomkins, C., Kumar, S., Orlicz, G., and Prestridge, K. "An experimental investigation of mixing mechanisms in shock-accelerated flow," *Journal of Fluid Mechanics*, 611:131-150, May 2008.

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